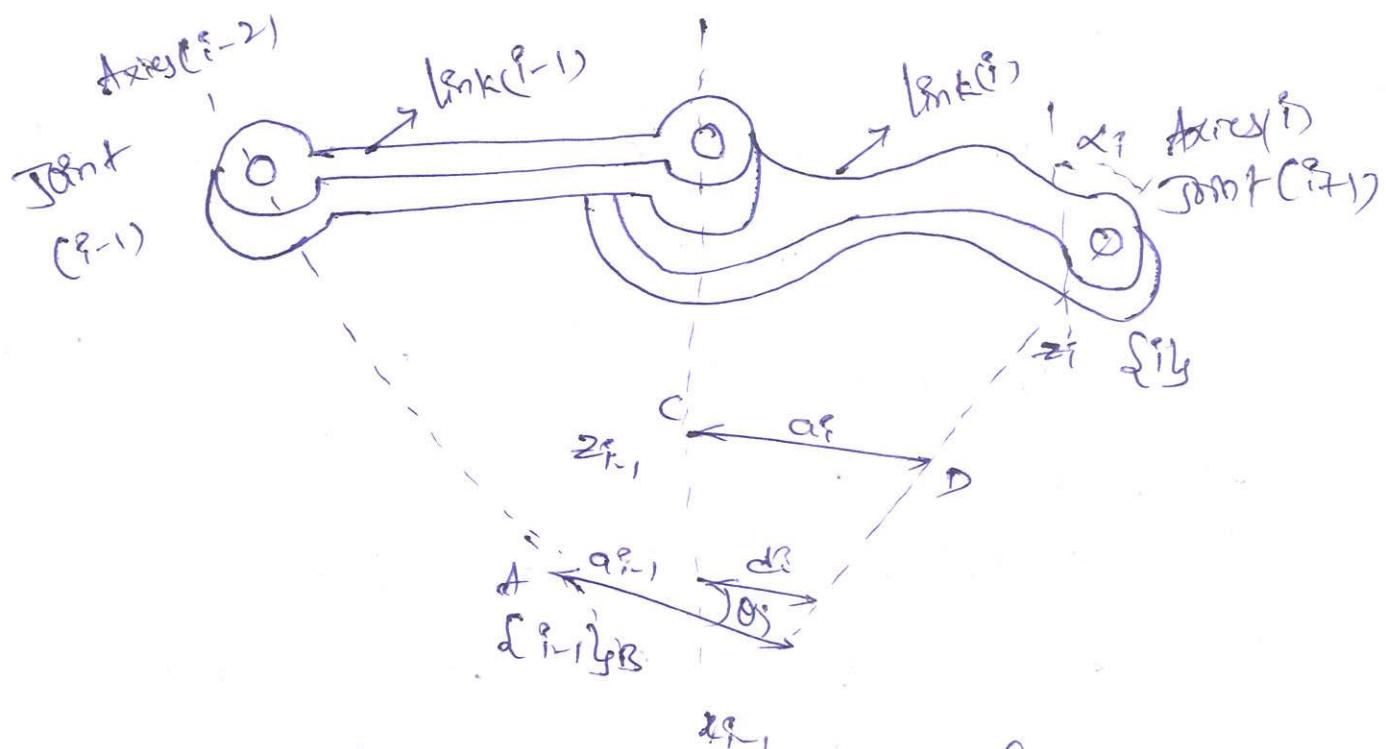


Unit - III

Manipulator Kinematics

Denavit-Hartenberg Notations:-



w.r.t. The above Schematic picture they are u DA-
parameters

2are link parameters (a_i, d_i)

2are joint parameters (θ_i, ϕ_i)

Link length (a_i):-

it is the distance b/w x_i -axis to the
inter section of z_{i-1} axes

Link Twist (α_i) :-

IT IS THE ANGLE b/w z_{i-1} axis and
 z_i axis which is measured about x_i -
axes

Link distance :- (D_i)

It is the distance b/w point q, later section (B) to the axis z_{e_i} , at the point 'C'.

Link angle (θ_i) :-

It is an angle b/w the z_{e_i} -axis to x_i axis, w.r.t to the z_{e_i} axis

The basic Transformations from {i-1} frames.

4 frames are :-

- 1) Rotation about z_{e_i} -axis z_{e_i} by θ_i
- 2) Translation by d_i with z_{e_i} axis
- 3) Translation about x_i -axis α_i
- 4) Rotation about x_i -axis with d_i .

$${}^{i-1}T_i = T_z(\theta_i) T_z(d_i) \cdot T_x(\alpha_i) T_x(d_i)$$

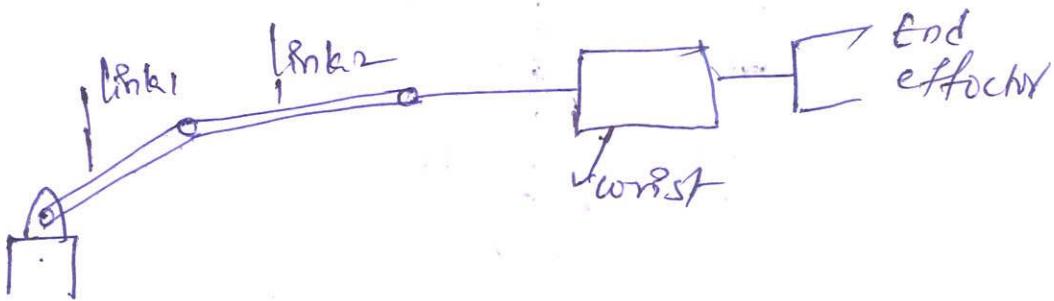
$$= \begin{bmatrix} R_z & D_i \\ 0 & 1 \end{bmatrix}$$

$${}^{i-1}T_i = \begin{pmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & d_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & a \cos\theta \\ \sin\theta & \cos\theta & 0 & a \sin\theta \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Direct Kinematics:-



In order to find the position and orientation of a n°- dof manipulator with the help of Joint link parameters then it is called Direct Kinematics (or) forward kinematics (or) direct kinematics

Inverse Kinematics

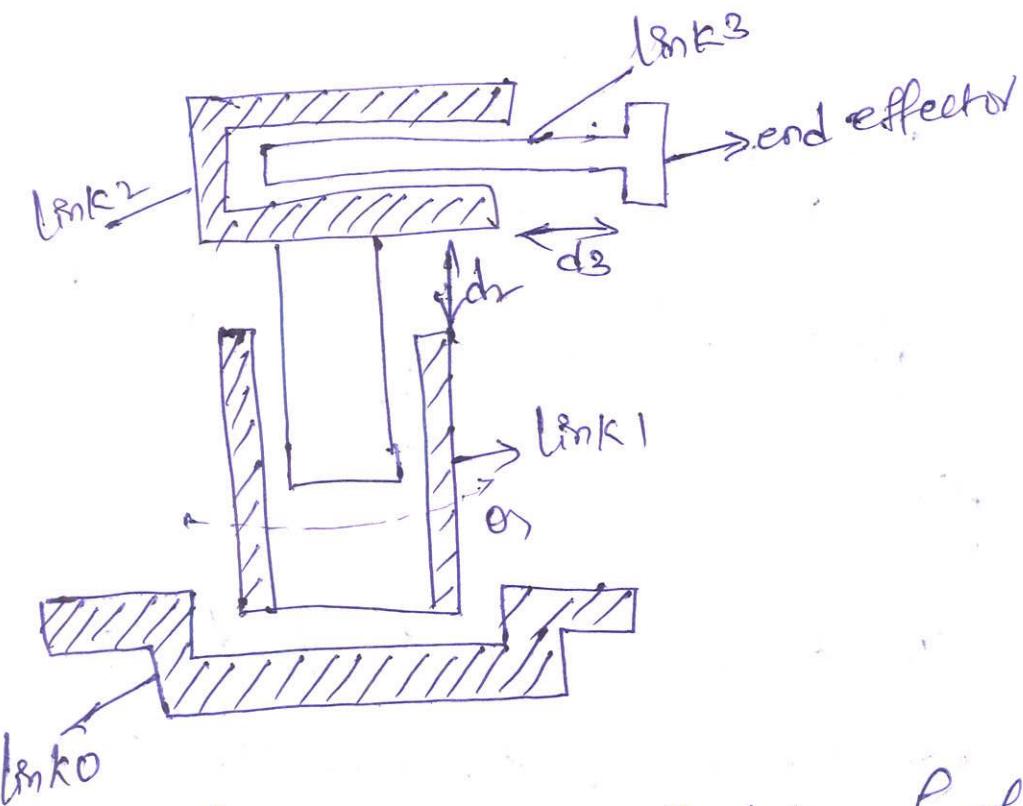
To find the inverse kinematics models with the help of position and orientation of n-Dof manipulator we can find the various joint link parameters

End effector Transformation matrix

$$\begin{pmatrix} n_x & o_x & a_x & dx \\ n_y & o_y & a_y & dy \\ n_z & o_z & a_z & dz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where
 a = Approach of the end effector
 O = Orientation
 n = Normal to (a, O)

problem:-



→ obtain the position and orientation for the above RPP 3-DOF manipulator. take also $\alpha_1 = 0, \alpha_2 = -90$

$$d_2 = 0$$

Joint-link parameter:-

| link _i | a_i | α_i | d_i | θ_i | replacement variable (q _i) | C_{θ_i} | S_{θ_i} | $S_{\theta_i} C_{\theta_i}$ | C_{θ_i} |
|-------------------|-------|------------|-------|------------|---|----------------|----------------|-----------------------------|----------------|
| 1 | 0 | 0 | 0 | θ_1 | θ_1 | C_{θ_1} | S_{θ_1} | 0 | 1 |
| 2 | 0 | -90 | d_2 | 0 | d_2 | 1 | 0 | -1 | 0 |
| 3 | 0 | 0 | d_3 | 0 | d_3 | 1 | 0 | 0 | 1 |

$$OT_3 = OT_1 \cdot 1T_2 \cdot 2T_3$$

DHT Notation:-

$$= \begin{bmatrix} \text{Cos}\theta_1 & -\text{Sin}\theta_1 \text{Cos}\alpha_1 & \text{Sin}\theta_1 \text{Sin}\alpha_1 & \text{d}_1 \text{Cos}\alpha_1 \\ \text{Sin}\theta_1 & \text{Cos}\theta_1 \text{Cos}\alpha_1 & -\text{Cos}\theta_1 \text{Sin}\alpha_1 & \text{d}_1 \text{Sin}\alpha_1 \\ 0 & \text{Sin}\alpha_1 & \text{Cos}\alpha_1 & \text{d}_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$OT_1(\theta_1) = \begin{bmatrix} \text{Cos}\theta_1 & -\text{Sin}\theta_1 & 0 & 0 \\ \text{Sin}\theta_1 & \text{Cos}\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$1T_2(\theta_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$2T_3(\theta_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Overall Transformation matx.

$$OT_3 = OT_1 \cdot 1T_2 \cdot 2T_3$$

$$\alpha_3 = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ \sin\theta & 0 & -\cos\theta & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & 0 & \sin\theta & d_3 \sin\theta \\ -\sin\theta & 0 & -\cos\theta & -d_3 \cos\theta \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Compare with The end effector matrix

$$\begin{bmatrix} nx & ox & ax & dx \\ ny & oy & ay & dy \\ nz & oz & az & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & d_3 \sin\theta \\ \sin\theta & 0 & -\cos\theta & -d_3 \cos\theta \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$nx = \cos\theta$$

$$ny = \sin\theta$$

$$nz = 0$$

$$ox = 0$$

$$oy = 0$$

$$oz = 1$$

$$ax = \sin\theta$$

$$ay = -\cos\theta$$

$$az = 0$$

$$dx = \sin\theta d_3$$

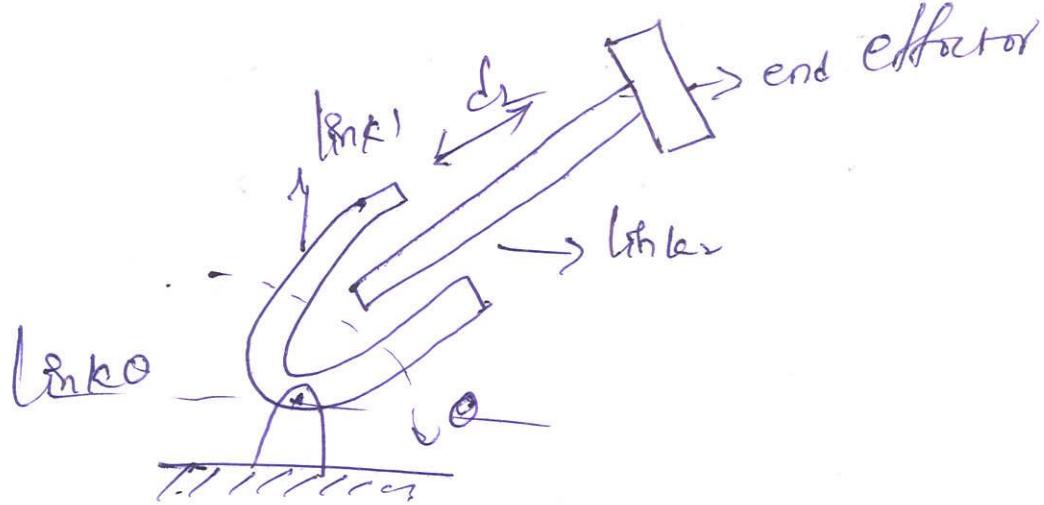
$$dy = -d_3 \cos\theta$$

$$dz = d_2$$

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unit-3, page-6/9

②



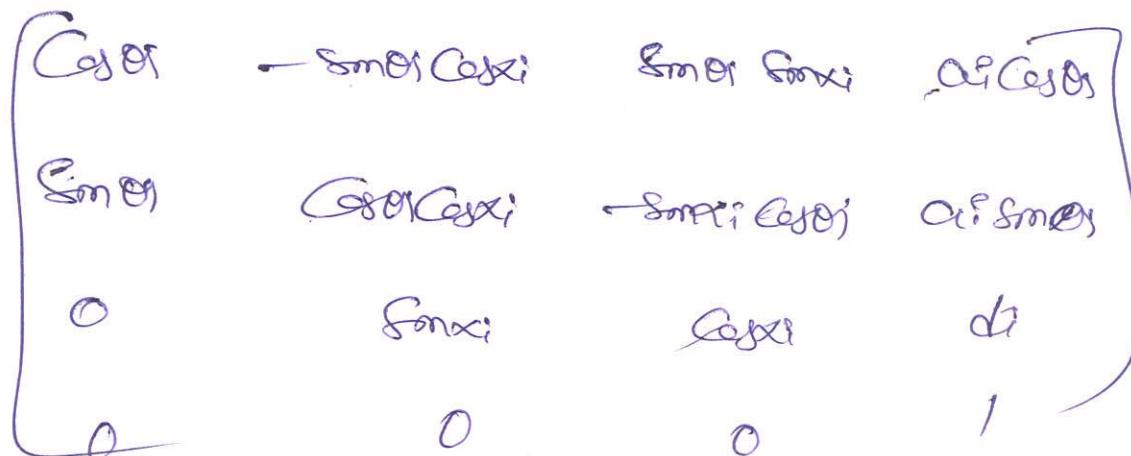
find the position and orientation for the help of above 2-DOF of a planar manipulator

Joint-link parameters:-

| link i | a _i | x _i | d _i | θ _i | q _i | Cosθ _i | sinθ _i | sinx _i | cosx _i |
|--------|----------------|----------------|----------------|----------------|----------------|-------------------|-------------------|-------------------|-------------------|
| 1 | 0 | 90 | 0 | 0 | 0 | Cosθ ₁ | sinθ ₁ | -1 | 0 |
| 2 | 0 | 0 | d ₂ | 0 | d ₂ | 1 | 0 | 0 | 1 |

$$O T_2 = O T_1 \cdot 1 T_2$$

D-H notation:-



$$O\Gamma_1 = \begin{bmatrix} \cos\theta & 0 & \sin\theta & \alpha \cos\theta \\ \sin\theta & 0 & -\cos\theta & \alpha \sin\theta \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$I\Gamma_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & \sin\theta & \alpha \cos\theta \\ \sin\theta & 0 & -\cos\theta & \alpha \sin\theta \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & \sin\theta & \alpha \cos\theta \\ \sin\theta & 0 & -\cos\theta & \alpha \sin\theta \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Unit -3, page -8) q

$$O\Gamma_2 = O\Gamma_1 \cdot \Gamma_{T_2}$$

Compare with the end effector matrix

$$\alpha_x = \cos \theta_1$$

$$\alpha_y = -\sin \theta_1$$

$$\alpha_z = 0$$

$$\alpha_{xx} = \sin \theta_1$$

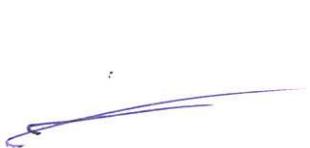
$$\alpha_{xy} = \cos \theta_1$$

$$\alpha_{xz} = 0$$

$$\alpha_{dx} = \sin \theta_1 d_1$$

$$\alpha_{dy} = d_1 \cos \theta_1$$

$$\alpha_{dz} = 0$$



Unit - 3, page - 9/9

